1-bit Hamming Compressed Sensing

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Abstract—Compressed sensing (CS) and 1-bit CS cannot directly recover quantized signals preferred in digital systems and require time consuming recovery. In this paper, we introduce 1-bit Hamming compressed sensing (HCS) that directly recovers a k-bit quantized signal of dimension \( n \) from its 1-bit measurements via invoking \( n \) times of Kullback-Leibler divergence based nearest neighbor search. Compared to CS and 1-bit CS, 1-bit HCS allows the signal to be dense, takes considerably less (linear and non-iterative) recovery time and requires substantially less measurements. Moreover, 1-bit HCS can accelerate 1-bit CS recover. We study a quantized recovery error bound of 1-bit HCS for general signals. Extensive numerical simulations verify the appealing accuracy, robustness, efficiency and consistency of 1-bit HCS.

I. INTRODUCTION

Recent results in compressed sensing (CS) [1][2][3] prove that a sparse or compressible signal can be exactly recovered from its linear measurements, rather than uniform samplings, at a rate significantly lower than the Nyquist rate. The measurement matrix is required to have the restricted isometry property (RIP) [4][5], or to be incoherent with the bases on which the signal is sparsely represented, for the purpose of ensuring the exact reconstruction via an \( \ell_p \) (\( 0 \leq p < 2 \)) penalized/constrained minimization of the measurement error. Random matrix is desirable in CS for its incoherence and RIP.

However, CS [6] encounters several problems when applied to practical digital systems, where analog-to-digital converters (ADCs) not only sample, but also quantize each measurement to a finite number of bits. One key problem is that CS cannot handle the quantized measurements (rounding of coarse quantization leads to large error, while fine quantization requires expensive ADCs). Thus 1-bit CS [7][8] is developed to reconstruct sparse signals from 1-bit measurements, which capture the signs of the CS measurements. The 1-bit measurements significantly reduce the costs and strengthen the robustness of hardware implementation. Although the 1-bit measurements lead to the loss of scale information, 1-bit CS ensures consistent reconstructions of signals on the unit \( \ell_2 \) sphere [9][10]. Similar to RIP in CS, the binary \( \ell_p \)-stable embedding (BSE) [8] of the 1-bit measurement guarantees the theoretical reconstruction and robustness.

Another important problem is that digital systems prefer to use the quantized recovery of the original signal, which they can directly process, but the recoveries of both CS [11] and 1-bit CS are real-valued. In order to apply them to digital systems, additional quantization is required. Moreover, the time consuming optimization based and iterative recovery in CS and 1-bit CS limits their applications in practical systems, especially when signals are of high-dimension. In addition, we prefer to control the trade-off between speed and accuracy in signal recovery. Quantized recovery offers a possible solution, because the number of bits in quantization can be adjusted to reach different resolutions. The last problem is that CS or 1-bit CS achieves exact recovery under the Nyquist rate due to replacement of the previous uniform sampling with random linear measurements or their signs. However, the signal is restricted to be sparse. Quantization is a coarse and irreversible description of the original signal, and to recover it is the same as recovering a box constraint. Thus it is possible to recover the quantization of a dense signal from a small number of measurements. In this paper, we mainly consider the problem of fast quantized recovery of a general signal.

The primary contribution of this paper is developing 1-bit Hamming compressed sensing (HCS) to recover the quantized signal from its 1-bit measurements with extremely small time cost and without signal sparsity constraint. In compression, we adopt the 1-bit measurements [8] to guarantee consistency (cf. longer version) and BSE but employ them in a different way. In particular, we introduce a bijection between each dimension of the signal and a Bernoulli distribution. The underlying idea of 1-bit HCS is to estimate the Bernoulli distribution for each dimension from the 1-bit measurements, and thus each dimension of the signal can be recovered from the corresponding Bernoulli distribution. In recovery, we propose a k-bit HCS quantizer for the signal domain, whose intervals are the mappings of the uniform linear quantization boundaries for the Bernoulli distribution domain. 1-bit HCS searches the nearest neighbor of the estimated Bernoulli distribution among the boundaries and recovers the quantization of the corresponding dimension as the HCS quantizer interval associated with the nearest neighbor. The main significance of 1-bit HCS is as follows: 1) it provides a direct and simple recovery of quantized signal for digital systems; 2) it only requires to compute \( nk \) Kullback-Leibler (KL) divergences for obtaining k-bit recovery of an \( n \)-dimensional signal, and is therefore considerably more efficient than CS and 1-bit CS; 3) successful recovery can be obtained from only \( O(\log n) \) measurements. Thus 1-bit HCS can be applied to general signals without sparse assumption. We theoretically study a quantized recovery error bound of 1-bit HCS by investigating the precision of the estimation and its impact on the KL divergence based nearest neighbor search. An HCS extension “compressed labeling” [12] significantly reduces the complexity of multi-label learning problem.

II. 1-BIT MEASUREMENTS

1-bit HCS recovers the quantized signal directly from its quantized measurements, each of which is composed of a finite number of bits. We consider the extreme case of 1-bit measurements of a signal \( x \in \mathbb{R}^n \), which are given by

\[
y = A(x) = \text{sign} \left( \Phi x \right),
\]

where \( \text{sign}(\cdot) \) is an element-wise sign operator and \( A(\cdot) \) maps \( x \) from \( \mathbb{R}^n \) to the Boolean cube \( \mathbb{B}^M := \{-1, 1\}^M \). Since the scale of the signal is lost in 1-bit measurements \( y \) (multiplying \( x \) with a positive scalar will not change the signs of the measurements), the consistent reconstruction can be obtained by enforcing the signal \( x \in \Sigma_k := \{ x \in S^{n-1} : \|x\|_0 \leq K \} \) where \( S^{n-1} := \{ x \in \mathbb{R}^n : \|x\|_2 = 1 \} \) is the \( n \)-dimensional unit hyper-sphere.
A. Bijection

In contrast to CS and 1-bit CS, 1-bit HCS does not recover the original signal, but reconstructs the quantized signal by recovering each dimension in isolation. In particular, according to Lemma 3.2 in [13], we show that there exists a bijection (cf. Theorem 1) between each dimension of the signal $x$ and a Bernoulli distribution, which can be uniquely estimated from the 1-bit measurements. The underlying idea of 1-bit HCS is to estimate the Bernoulli distribution for each dimension, and recover the quantization of the corresponding dimension as the interval where the Bernoulli distribution’s mapping lies in.

**Theorem 1. (Bijection)** For a normalized signal $x \in \mathbb{R}^n$ with $\|x\|_2 = 1$ and a normalized Gaussian random vector $\phi$ that is drawn uniformly from the unit $\ell_2$ sphere in $\mathbb{R}^n$ (i.e., each element of $\phi$ is firstly drawn i.i.d. from the standard Gaussian distribution $\mathcal{N}(0, 1)$ and then $\phi$ is normalized as $\|\phi\|_2$), given the $i^{th}$ dimension of the signal $x_i$ and the corresponding coordinate unit vector $e_i = \{0, \ldots, 0, 1, 0, \ldots, 0\}$, where 1 appears in the $i^{th}$ dimension, there exists a bijection $P : \mathbb{R} \rightarrow \mathbb{P}$ from $x_i$ to the Bernoulli distribution of the binary random variable $s_i = \text{sign}(\langle x_i, \phi \rangle) \cdot \text{sign}(\langle e_i, \phi \rangle)$:

$$P(x_i) = \left\{ \begin{array}{ll}
\Pr(s_i = -1) = \frac{1}{2} \arccos(x_i), \\
\Pr(s_i = 1) = 1 - \frac{1}{2} \arccos(x_i). \end{array} \right.$$  

Since the mapping between $x_i$ and $P(x_i)$ is bijective, given $P(x_i)$, the $i^{th}$ dimension of $x$ can be uniquely identified. According to the definition of $s_i$, $P(x_i)$ can be estimated from the instances of the random variable $\text{sign}(\langle x, \phi \rangle) \cdot \text{sign}(\langle e_i, \phi \rangle)$ (i.e., $\text{sign}(\langle e_i, \phi \rangle)$ is also used but does not depend on $x$), which are exactly the 1-bit measurements $y$ defined in (1). Therefore, the 1-bit measurements $y$ include sufficient information to reconstruct $x_i$ from the estimation of $P(x_i)$, and the recovery accuracy of $x_i$ depends on the accuracy of the estimation to $P(x_i)$.

Given a signal $x$, its quantization $q = Q(x)$ by HCS quantizer $Q(\cdot)$, and the quantized recovery $q^* = R(y)$ obtained by 1-bit HCS reconstruction $R(\cdot)$ from the 1-bit measurements $y = A(x)$, $err_H$ is the quantization error determined by the difference between $q$ and $q^*$. The upper bound of $err_H$ will be given in Sections 4.

### III. k-bit Reconstruction

The primary contribution of this paper is the quantized recovery in 1-bit HCS, which reconstructs the quantized signal from its 1-bit measurements (1). Figure 1(a) illustrates 1-bit HCS quantized recovery. To define the HCS quantizer, we firstly find $k$ boundaries $P_j(j = 0, \ldots, k - 1)$ (4) in Bernoulli distribution domain by imposing the uniform linear quantizer to the range of $P_j^-$. Given an arbitrary $x_i$, the nearest neighbor of $P(x_i)$ among the $k$ boundaries $P_j(j = 0, \ldots, k - 1)$ indicates the interval $q_i$ that $x_i$ lies in the signal domain. The $k + 1$ boundaries $S_j(j = 0, \ldots, k)$ associated with the $k$ intervals $q_j(j = 0, \ldots, k)$ are calculated from the $k$ boundaries $P_j(j = 0, \ldots, k - 1)$ according to the bijection defined in Theorem 1. In 1-bit HCS recovery, $P(x_i)$ is estimated as $P(x_i)$ from the 1-bit measurements $y$. Then the nearest neighbor of $P(x_i)$ among the $k$ boundaries $P_j(j = 0, \ldots, k - 1)$ is determined by comparing the KL-divergences between $P(x_i)$ and $P_j$. The quantization of $x_i$ defined by HCS quantizer is recovered as the interval $q_i$ corresponding to the nearest neighbor.

In this section, we first introduce the HCS quantizer, which is a mapping resulting from the uniform linear quantizer of the Bernoulli distribution domain to the signal domain. The quantized recovery procedure is composed of $n$ times of KL-divergence based nearest neighbor searches. Thus it is a linear algorithm much faster than the conventional reconstruction algorithms of CS and 1-bit CS, which require optimization with the $\ell_p$ ($0 \leq p < 2$) constraint/penalty, or iterative thresholding/greedy search. We then study the upper bound of the quantized recovery error $err_H$.

#### A. HCS quantizer

Since 1-bit HCS aims at recovering the quantization of the original signal, we firstly introduce HCS quantizer, which defines the intervals and boundaries for quantization in the signal domain. These intervals and boundaries are uniquely derived from a predefined uniform linear quantizer in the Bernoulli distribution domain. Given a signal and the boundaries of HCS quantizer, its $k$-bit quantization can be identified. We will show HCS quantizer performs closely to the uniform linear quantizer.

Note that in the quantized recovery of 1-bit HCS, the reconstruction and quantization are simultaneously accomplished. Thus the HCS quantizer will not play an explicit role in the recovery procedure. However, it is related to and uniquely determined by the quantization of the Bernoulli distribution domain, which plays an important role in the recovery and explains the reconstruction $q^*$. Moreover, it will be applied to the error bound analyses for $err_H$.

We introduce the HCS quantizer $Q(\cdot)$ by defining a bijective mapping from the boundaries of the Bernoulli distribution domain to the intervals of the signal domain according to Theorem 1. Assume the range of a signal $x$ is given by:

$$-1 \leq x_{inf} \leq x_i \leq x_{sup} \leq 1, \forall i, \ldots, n.$$  

By applying the uniform linear quantizer with the quantization interval $\Delta$ to the Bernoulli distribution domain, we get the corre-
The quantization of $x_i$ can then be recovered by searching the nearest neighbor of $\hat{P}(x_i)$ among the $k$ boundary Bernoulli distributions $P_j (j = 0, \ldots, k-1)$ in (4). In this paper, the distance between $P_j$ and $\hat{P}(x_i)$ is measured by the KL-divergence:

$$D_{KL} \left( P_j || \hat{P}(x_i) \right) = P_j^y \log \frac{P_j^y}{\hat{P}(x_i)^y} + P_j^{\overline{y}} \log \frac{P_j^{\overline{y}}}{\hat{P}(x_i)^{\overline{y}}},$$

$$\forall i = 1, \ldots, n, \forall j = 0, \ldots, k-1.$$

(9)

The interval that $x_i$ lies in among the $k$ intervals defined by the boundaries $S_j (j = 0, \ldots, k)$ in (6) is identified as the one whose corresponding boundary distribution $P_j$ is the nearest neighbor of $\hat{P}(x_i)$. Therefore, the quantized recovery of $x$, i.e., $q^*$, is given by

$$R(y) = q^*, q_i^* = 1 + \arg \min D_{KL} \left( P_j || \hat{P}(x_i) \right),$$

$$\forall i = 1, \ldots, n, \forall j = 0, \ldots, k-1.$$

(10)

Thus the interval that $x_i$ lies in can be recovered as

$$S_{q_i^*-1} \leq x_i \leq S_{q_i^*}.$$

(11)

The 1-bit HCS recovery algorithm is fully summarized in (10), which only includes simple computations without iteration and thus can be easily implemented in real systems. According to (10), the quantized recovery in 1-bit HCS requires $nk$ computations of KL-divergence between two Bernoulli distributions. This indicates the high efficiency of 1-bit HCS (linear recovery time), and the trade-off between resolution ($k$) and time cost ($nk$).

C. Quantized recovery error bound

We investigate the error bound of the quantized recovery (10) by studying the difference between $q_i$ and $q_i^*$, which are the quantization of $x_i$ and its quantized recovery by 1-bit HCS, respectively. The difference between $q$ and $q^*$ defines the error $\epsilon_{RH}$, which is the error caused by 1-bit HCS reconstruction (10):

$$[\epsilon_{RH}] = \begin{cases} 
S_{q_i} - S_{q_i^*+1} \leq (q_i - q_i^* - 1) \Delta_{\max}, & q_i > q_i^*; \\
0, & q_i = q_i^*; \\
S_{q_i^*+1} - S_{q_i} \leq (q_i^* - q_i - 1) \Delta_{\max}, & q_i < q_i^*.
\end{cases}$$

(12)

The $\Delta_{\max}$ denotes the largest interval between neighboring boundaries of the HCS quantizer, i.e., $\Delta_{\max} = \max_{j=1, \ldots, k} (S_j - S_{j-1})$.

In order to investigate the difference between $q_i$ and $q_i^*$, we study the upper bound for the probability of the event that the true quantization of $x_i$ is $q_i = 1 + \alpha$, while its recovery by 1-bit HCS is $q_i^* = 1 + \beta(\beta \neq \alpha)$. According to the HCS quantizer (7) and the 1-bit HCS reconstruction (10), this probability is

$$\Pr \left( \beta = \arg \min_j D_{KL} \left( P_j || \hat{P}(x_i) \right) \mid S_0 \leq x_i \leq S_{a+1} \right).$$

(13)

In order to study the conditional probability in (13), we first consider an equivalent event of $\beta = \arg \min_j D_{KL} \left( P_j || \hat{P}(x_i) \right)$, shown in the following Lemma 1.

**Lemma 1. (Equivalence)** The event that the nearest neighbor of $\hat{P}(x_i)$ among $P_j (j = 0, \ldots, k-1)$ is $P_{j_0}$ equals to the event that $\hat{P}(x_i)$ is closer to $P_{j_0}$ than both $P_{j_0-1}$ and $P_{j_0+1}$, where the distance between $P_j$ and $\hat{P}(x_i)$ is measured by KL divergence (9), i.e.,

$$\beta = \arg \min_j D_{KL} \left( P_j || \hat{P}(x_i) \right) \iff$$

$$\begin{cases} 
D_{KL} \left( P_{j_0-1} || \hat{P}(x_i) \right) - D_{KL} \left( P_{j_0} || \hat{P}(x_i) \right) > 0, \\
D_{KL} \left( P_{j_0+1} || \hat{P}(x_i) \right) - D_{KL} \left( P_{j_0} || \hat{P}(x_i) \right) > 0.
\end{cases}$$

(14)
The conditional probability given in (13) can be upper bounded by two other conditional probabilities, whose conditions are the two cases of the condition in (13).

**Corollary 1. (Upper bounds in two cases)** The conditional probability given in (13) can be upper bounded by

$$\Pr \left( \beta = \arg \min_j D_{KL} \left( P_j \| \hat{P}(x_i) \right) \mid S_\alpha \leq x_i \leq S_{\alpha+1} \right) \leq \begin{cases} 
\Pr \left( D_{KL} \left( P_{\beta-1} \| \hat{P}(x_i) \right) - D_{KL} \left( P_\beta \| \hat{P}(x_i) \right) > 0 \mid S_\alpha \leq x_i \leq S_{\alpha+1} \leq S_\beta \right), \\
\Pr \left( D_{KL} \left( P_{\beta+1} \| \hat{P}(x_i) \right) - D_{KL} \left( P_\beta \| \hat{P}(x_i) \right) > 0 \mid S_{\beta+1} \leq S_\alpha \leq x_i \leq S_{\alpha+1} \right). 
\end{cases}$$

Hence we bound the conditional probability in (13) by exploring the upper bounds of the two conditional probabilities in Corollary 1.

**Proposition 1. (Two probabilistic bounds)** The two conditional probabilities in (15) are upper bounded by

$$\Pr \left( D_{KL} \left( P_{\beta-1} \| \hat{P}(x_i) \right) - D_{KL} \left( P_\beta \| \hat{P}(x_i) \right) > 0 \mid S_\alpha \leq x_i \leq S_{\alpha+1} \leq S_\beta \right) \leq \frac{1}{2} \exp \left( -2m \cdot \left( \frac{1}{\pi} \arccos (x_i) - \frac{1}{f \left( P_\beta \right) + 1} \right)^2 \right), \tag{16}$$

$$\Pr \left( D_{KL} \left( P_{\beta+1} \| \hat{P}(x_i) \right) - D_{KL} \left( P_\beta \| \hat{P}(x_i) \right) > 0 \mid S_{\beta+1} \leq S_\alpha \leq x_i \leq S_{\alpha+1} \right) \leq \frac{1}{2} \exp \left( -2m \cdot \left( \frac{1}{f \left( P_{\beta+1} \right) + 1} - \frac{1}{\pi} \arccos (x_i) \right)^2 \right), \tag{17}$$

where \( f \) is defined as

$$f \left( P_j \right) = \left( \frac{\left(P_j - \frac{1}{2}\right) \left(1-P_j \right) \left(1-P_j\right)^{1-P_j}}{\left(P_j - \Delta\right) \left(P_j - \Delta\right)^{1-P_j}} \right)^{1/\Delta}. \tag{18}$$

By using Lemma 1, Corollary 1 and Proposition 1, we have the following Theorem about the upper bound of the probability in (13).

**Theorem 2. (Quantized recovery bound)** Given HCS quantizer \( Q(\cdot) \) in (7) and 1-bit HCS reconstruction \( R(\cdot) \) in (10), the probability of the event that the true quantization of \( x_i \) is \( q_i = 1 + \beta(q_i \neq q_i^*) \) is upper bounded by

$$\Pr \left( [R(y)]_i = q_i^* \mid [Q(x)]_i = q_i \right) = \Pr \left( \beta = \arg \min_j D_{KL} \left( P_j \| \hat{P}(x_i) \right) \mid S_\alpha \leq x_i \leq S_{\alpha+1} \right) \leq \begin{cases} 
\frac{1}{2} \exp \left( -2m \cdot \left( \frac{1}{f \left( P_{\alpha+1} \right) + 1} - \frac{1}{\pi} \arccos (x_i) \right)^2 \right), \\
q_i > q_i^*; \\
\frac{1}{2} \exp \left( -2m \cdot \left( \frac{1}{\pi} \arccos (x_i) - \frac{1}{f \left( P_{\alpha+1} \right) + 1} \right)^2 \right), \\
q_i < q_i^*. 
\end{cases} \tag{19}$$

The minimum amount of 1-bit measurements that ensures the successful quantized recovery in 1-bit HCS is then directly obtained from Theorem 2.

**Corollary 2. (Amount of measurements)** 1-bit HCS successfully reconstructs \( x_i \) with probability exceeding \( 1 - \eta \) \((0 \leq \eta \leq 1)\) if the
number of measurements

\[ m \geq \frac{1}{2\delta_i} \log \frac{1}{2\eta}, \]  

(20)

where

\[ \delta_i = \min \left[ \left( \frac{1}{f(P_{q_i})} + 1 \right) - \frac{1}{2} \arccos (x_i) \right]^2, \]

\[ \left( \frac{1}{\pi} \arccos (x_i) - \frac{1}{f(P_{q_i})} + 1 \right)^2 \]  

(21)

Moreover, 1-bit HCS successfully reconstructs the signal \( x \) with probability exceeding \( 1 - \eta \) if the number of measurements

\[ m \geq \frac{1}{2 \min_1 \delta_i} \log \frac{n}{2\eta}, \]  

(22)

Remark: Corollary 2 states that the quantization of an \( n \)-dimensional signal \( x \) on the unit sphere can be successfully recovered by 1-bit HCS from \( m = O(\log n) \) with high probability. Thus 1-bit HCS provides an economical sampling scheme that does not rely on sparse or compressible assumption to the signal.

A new issue in 1-bit HCS is the trade-off between the measurement amount \( m \) and the recovery resolution \( k \). According to the definition of \( \delta_i \) in (21), both the upper bound for the probability of reconstruction failure in (19) and the least number of measurements ensuring reconstruction success in (20) will be reduced if \( |q_i - q_i^*| \) increases. This indicates two facts: 1) the interval \( \delta_i \) is in is easier to be mistakenly recovered as its nearest intervals; and 2) when we increase the number of bits \( k \) in quantized recovery, \( x_i \) will become closer to the boundaries \( S_{q_i-1} \) and \( S_{q_i} \), which leads to the decreasing of \( \min_1 \delta_i \) in (22). In this case, the number of measurements \( m \) has to be increased in order to ensure a successful recovery.

IV. Empirical study

This section evaluates 1-bit HCS and compares it with “BIHT [8] (for 1-bit CS)+HCS quantizer” on two groups of numerical experiments. We use average quantized recovery error \( \sum_i |q_i - q_i^*| / nk \) to measure errH shown in Section 3.3. In each trial, we draw a normalized Gaussian random matrix \( \Phi \in \mathbb{R}^{m \times n} \) given in Theorem 1 and a signal of length \( n \) and cardinality \( K \), whose \( K \) nonzero entries are drawn uniformly at random on the unit \( \ell_2 \) sphere. Please refer to the supplementary material for complete experiments.

A. Phase transition in the noiseless case

We first study the phase transition properties of 1-bit HCS and 1-bit CS on quantized recovery error and on recovery time in the noiseless case. We conduct 1-bit HCS and “BIHT+HCS quantizer” for \( 10^3 \) trials. In particular, given fixed \( n \) and \( k \), we uniformly choose 100 different \( K/n \) values between 0 and 1, and 100 different \( n/m \) values between 0 and 4. For each \( \{K/n, m/n\} \) pair, we conduct 10 trials, i.e., 1-bit HCS recovery and “1-bit CS+HCS quantizer” of 10 \( n \)-dimensional signals with cardinality \( K \) from their \( m \) 1-bit measurements. The average quantized recovery errors and average time costs of the two methods on overall \( 10^3 \) \( \{K/n, m/n\} \) pairs are shown in Figure 2.

In Figure 2, the phase plots of quantized recovery error show the quantized recovery of 1-bit HCS is accurate if the 1-bit measurements are sufficient. Compared to “1-bit CS+HCS quantizer”, 1-bit HCS needs slightly more measurements to reach the same recovery precision, because 1-bit CS recovers the exact signal, while 1-bit HCS recovers its quantization. This is an unavoidable price for direct recovery of quantization. However, the phase plots of quantized recovery time shows that 1-bit HCS takes substantially less time than “1-bit CS+HCS quantizer”. Thus 1-bit HCS can significantly improve the efficiency of practical digital systems and eliminate the hardware cost for additional quantization. So the trade-off is quite advantageous.

B. Quantized recovery error vs. measurements in the noisy case

We show the trade-off between quantized recovery error and the amount of measurements on 2500 trials for noisy signals of different \( n, K \), \( k \) and signal-to-noise ratio (SNR). Given fixed \( n, K, k \) and SNR, we uniformly choose 50 values of \( m \) between 0 and 16\( m \). For each \( m \) value, we conduct 50 trials of 1-bit HCS recovery and “1-bit CS+HCS quantizer” by recovering the quantizations of 50 noisy signals from their \( m \) 1-bit measurements. The quantized recovery error and time cost of each trial are shown in Figure 3.

Figure 3 shows that the quantized recovery error of both 1-bit HCS and “1-bit CS+HCS quantizer” drops drastically with an increase in the number of measurements. For dense signals with large noise, the two methods perform nearly the same on the recovery accuracy. This indicates that 1-bit HCS works well on dense signals and is robust to noise compared to CS and 1-bit CS. In addition, the time taken for 1-bit HCS increases substantially slower than that of “1-bit CS+HCS quantizer” with an increase in the number of measurements.

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